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ABSTRACT

Arguing that high attrition rates and low grades in trigonometry classes underscore the need for changes in the approach to trigonometry instruction, these papers suggest an instructional model and present a proposal for its adoption at the College of San Mateo (CSM). The first paper begins with the argument that students' lack of preparation for the study of trigonometry makes a math lab approach, combined with a regular lecture series, an appropriate way to reinforce math experiences through exercises. Next, a proposed four-unit course, comprising two distinct half-semester, two-unit courses, is described in terms of its format, objectives, instructional techniques, and emphases. Then, the paper outlines the major differences between the proposed class and current procedures, focusing on mandatory lab attendance, tutorial staff roles, diagnostic testing, and blocking out the same hour every day of the week for the class. Appendices provide a longitudinal look at CSM trig enrollments, and a curriculum guide and schedule for the proposed trigonometry class. The second paper presents a proposal for adopting the four-unit course with three lecture hours and two hours of problem solving per week at CSM. A rationale for restructuring trigonometry at CSM is followed by a discussion of course format, an explanation of the functions of the lecture and problem-solving sessions, and an analysis of the effects of the proposal on personnel. Finally, a paper is presented examining the math deficiencies of trigonometry students. (LAL)

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SOME REASONS FOR A NEED TO CHANGE THE APPROACH TO TRIGONOMETRY INSTRUCTION AND A PROPOSAL FOR CHANGE

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SOME REASONS FOR A NEED TO CHANGE THE APPROACH

TO TRIGONOMETRY INSTRUCTION

Trigonometric functions provide a departure for students from the customary functions of algebra. About the only special function encountered in Intermediate Algebra is the absolute value function. And so students come to Trigonometry having learned the fundamental operations on and properties of polynomial and rational functions in the previous course. Even so, they come woefully ill-prepared for the study of trigonometric functions.

Each trig function is, in a sense, a unary operator and thus a relatively new experience for the student. This fact, coupled with many students' inadequate algebraic skills and convictions, seems to cause the student to be mired in difficulty in the trig course from the beginning. Examples in point are lack of understanding of the concept of function, of function domain and range and restrictions inherent in each.

The student, it would seem, needs to think his (her) way through more mathematical experiences via exercises. A lab situation, in combination with a regular lecture series, might provide extra reinforcement. A lab environment would motivate the student to do (or try to do) assigned problems and remedial work, if trained help were available for consultation and/or guidance. A math lab can provide the ideal environment for the student to overcome any "mystery", gain conviction about what is valid vis a vis what is invalid, get down to "brass tacks" and overcome identified inadequacies that hinder his (her) progress in the course.

Trigonometry is not a difficult subject and can be learned by the average student. Yet instructors encounter such high attrition in this course, as well as a struggle to the finish even by those who stick it out. Over the past 6 terms at CSM , the percentage of first census

enrollees who completed the course (including grades of D and F) was at best 66% and at worst 56% according to available data. (The supporting data is provided in Appendix A) & Appendix D).

It may be that insufficient exposure time to the subject is a primary cause of the heavy attrition experienced in the course, as a whole. The record drop rate reported in Appendix A merits study. Causes of the heavy attrition experienced in this course should be hypothesized and steps should be taken to test these hypotheses via lab implementation.

And even more alarming is the overall record of poor grades issued in trigonometry classes. Data to illustrate that fact are being collected. The phenomenon applies to no one instructor's trigonometry class, but is a universally perceived occurrence.

The following approach to solving this problem is proposed.

Course Designation

1. Math 130AB : A 4-unit course for enrollees, 3-unit load for the instructor.
   Curriculum outline appears in Appendix B.

2. Math 130A : A 2-unit course which begins at the semester's beginning and terminates at mid-semester. This course would satisfy the trig requirement for tech courses. Its inclusion as a separate course in the catalogue would enable students unable - or not wishing - to finish the course (at this time) to get two units credit. Its numeric designation would preclude their entering any math course requiring Math 130AB as a prerequisite.

3. Math 130B : The remainder of Math 130AB. Its designation would enable entry at mid-semester to those who might have dropped out in the previous semester, and thus complete the course for credit.
   Curriculum guide for Math 130B is indicated in Appendix B.

Course Format

The Holmgren Mode A is recommended (Appendix C)

1. Regular lecture method - 3 hours weekly, and in three days
2. Lab - 2 hours weekly - on those days when there is not a lecture.

The Holmgren Mode B (Appendix C) would require considerable time from the respective instructors to synchronize activities with the assigned tutors, if lab implementation were to be effective.

Course Objectives

1. To define the trigonometric functions, to study their properties, their graphs and the properties of their inverses.
2. To apply trigonometry to real-life problems
3. To learn how to solve triangles
4. To do scientific calculations

Some Techniques Recommended to Achieve Objectives

1. Math lab access

2. Each chapter would be preceded by a brief introduction that identifies what new topics are introduced and explains why they are important.

3. Each student will take a diagnostic test on techniques frequently used in the new chapter. Test is taken in the math lab before the first lecture on the chapter - thus reinforcing in the student's mind that he (she) is getting ready for the new topic(s). Tutor records student's score. If that score is deemed to be inadequate, tutor provides student with a reference to an algebra text with the necessary remedial material.

4. Student presents each assigned exercise to the tutor for grading and recording no later than second calendar day following date of assignment, excepting Friday assignments which are to be graded no later than the following Tuesday. A quiz follows each set of exercises turned in. Quiz is mandatory, is related to the completed exercises, and is part of the student's grade record. Quiz is taken in the lab, under the supervision of the tutor, and without assistance to the test taker.

5. Cumulative exams are taken at designated intervals in the course. A level of competency is determined by the instructor as being the minimum acceptable by him (her)

from students. Any student who does not meet this criterion must re-take the exam (a different form) after sufficient remediation has been done by him (her). Replacement of the initial grade by an improved grade is up to individual instructors' decision. There will be no more than two attempts per exam.

Course Emphasis

Objective 3 provides the least difficulty normally. Scientific calculations and calculator usage can be handled in relatively short time. These latter techniques will be resorted to so often that little difficulty will be encountered here.

The most difficult part of the course is applications and the understanding of functions and of their properties.

Problem-solving needs emphasis because of generally inadequate preparation in solving applied problems in previous mathematics courses. There is also that cultural fear of "story problems". However, this most important skill if of great value in calculus, physics, engineering, etc.

Objective 1 provides the greatest difficulty generally. Its understanding and mastery call for considerable skill in algebra, reasoning ability, skill in the ability to sort the learned information and apply it appropriately in proving identities and in solving conditional equations.

MAJOR DIFFERENCES IN THIS PROPOSAL AND CURRENT PROCEDURE IN TRIGONOMETRY CLASSES

1. This proposal requires mandatory attendance in lab with a tutor. It is recommended that at least 2 tutors be provided per section. Tutors will perform the following tasks.
   a. Record attendance
   b. Check homework
   c. Distribute and grade quizzes
   d. Record grades
   e. Monitor, correct and grade repeats on quizzes when repetition is warranted

f. Administer diagnostic test at outset of each chapter
g. Provide, upon request, remedial unit for students indicating deficiencies in diagnostic test.
h. Provide tutorial assistance

2. This proposal depends on securing tutorial staff help.

3. This proposal encompasses diagnostic testing, turning-in of homework; upon submission of homework, a quiz is to be taken during lab session. Quiz may be re-taken only once. Concepts from homework, and lecture material may constitute quiz content. Cumulative exams must be passed at or above the minimum criteria level established by the instructor. If this minimum level is not met, student must take another exam on such material.

4. The alternative Holmgren Mode B plan has been reproduced in Appendix C. Its variety counts upon careful synchronization between instructor and tutors, and reliance on a tutorial staff that would be super dependable.

5. The current proposal requires the blocking out of the same hour 5 days a week in program planning. While some students may not need to spend the entire two hours in the lab each week, a grading scheme might be constructed that would require students to solve certain difficult problems to earn an A in the course. Exercises provided might be on three levels of difficulty:

   a. routine exercises, with easy mechanics
   b. more difficult mechanics
   c. difficult mechanics or problems of a theoretical nature.

Conclusion

This proposal was written with the day school in mind. However, night school census data were included in the attrition data referred to in Appendix A. A way of dealing with this phenomenon in night and summer school is still to be sought.

One closing suggestion is to follow-up the pilot program if initiated, after one year. The Hawthorne Effect, not withstanding! The follow-up study might compare attrition for the experimental and control groups over the period.

Finally, even the Holmgren Mode A schedule presumes considerable synchronization between instructors of Math 130 and the tutors engaged to help the skippers.

APPENDIX A

TRIGONOMETRY ENROLLMENTS AT 1st CENSUS, 2nd CENSUS, AND END OF SEMESTER AT COLLEGE OF SAN MATEO : FALL, 1979 - FALL, 1981

FALL, 1979	1st Census	2nd Census		End of Semester	
Math 130 AA	41	26		22	
Math 130 AB	34	23		15	
Math 130 AC	27	20		12	
Math 130 AD	40	34		29	
Math 130 AE	20	16		13	
Math 130 KC	28	23	(75%)	16	(56% of 1st Cen)
	190	142		107	
SPRING, 1980					
Math 130 AB	33	--		--	
Math 130 AC	38			18	
Math 130 AD	30			15	
Math 130 AE	25			15	
Math 130 KA	38			17	
	164 - 33=131			65 (	(50% of 1st Census)
SUMMER, 1980					
Math 130 BB	35				
Math 130 BC	31				
Math 130 KB	40				
	106				
FALL, 1980					
Math 130 AA	37	31		29	
Math 130 AB	22	17		13	
Math 130 AC	34	30		23	
Math 130 AD	35	30		23	
Math 130 AE	20	20		15	
Math 130 KC	30	24	(85%)	14	(66% of 1st Cen)
	178	152		117	
SPRING, 1981					
Math 130 AA	31	27		21	
Math 130 AB	34	31		26	
Math 130 AC	40	35		27	
Math 130 AD	31	22		20	
Math 130 LA	34	23	(81%)	13	(63% of 1st Cen)
	170	138		107	
SUMMER, 1981					
Math 130 BB	34	23		21	
Math 130 BC	36	24		20	
	70	47	(67%)	41	(59% of 1st Cen)
FALL, 1981					
Math 130 AA	37	31		26	
Math 130 BB	40	30		24	
Math 130 EA	39	35		29	
Math 130 FC	37	28		20	
Math 130 GA	29	22		19	
Math 130 NC	46	35		32	
	228	181	(79%)	150	(64% of 1-st Cen)

APPENDIX B

CURRICULUM GUIDE FOR MATHEMATICS 130A, 130AB, 130B

I. TITLE AND SUBJECT:

Mathematics 130A, 130AB, 130B - Analytic Trigonometry
Three Lecture Hours per week and two lab hours per week.
Four semester units for Math 130AB
Two semester units for Math 130A
Two semester units for Math 130B

Prerequisites : Math 115 and Math 120 or 121 with grades of C or better, or high school preparation including 1½ years of Algebra and one year of Geometry with a grade of C or better. Math 130B prerequisite: completion of Math 130A with a grade of C or better.

II. AIMS AND OBJECTIVES:

A. To develop and to work with the fundamental principles of analytical plane trigonometry.
B. To understand the rationale of trigonometric and logarithmic tables,to become proficient in use of their successor, the trigonometric and log functions on the calculator.
C. To develop methods of valid proof.
D. To provide varied applications of trigonometry to the sciences, via problem solving.
E. To meet subject requirements of students in General Education and in special fields.

III. SCOPE OF COURSE

A. ReStatement of the Field Axioms
B. Relations, Functions, and Their Graphs
C. Trigonometric Functions
   1. Of Angles
   2. Of Real Numbers
D. Graphs of Trigonometric Functions
E. Solution of Triangles
   Vectors.
F. Trigonometric Identities
G. Trigonometric Equations
H. Inverse Trigonometric Functions and Their Graphs
I. Logarithmic and Exponential Functions
J. Complex Numbers
K. De Moivre's Theorem

IV. SCOPE OF MATH 130A : Topics A,B,C,D,E under III Above
SCOPE OF MATH 130B : Topics F,G,H,I,J,K under III Above
SCOPE OF MATH 130AB : All Topics Listed Under III Above

V. METHODS OF INSTRUCTION

Textbook Study and Problem Assignments; Lecture-Demonstration;

APPENDIX B : Continued

Lab Facility with Reference Materials - Texts, Tapes, Specific References for Remediation; Tutorial Assistance and Monitoring

V. FACILITIES

Standard Mathematics Classroom, with audio-visual facilities and chalkboard drafting instruments; Lab with reference materials - texts and tapes

VI. EVALUATION

Grading is based on homework, on quizzes related to homework, on cumulative examinations, and additional criteria determined by individual instructors

VII. TEXT

At the discretion of the instructor

APPENDIX C

THE HOLMGREN MODEL FOR MATH 130AB

MODE A

Monday	Tuesday	Wednesday	Thursday	Friday
Lecture 50 Min.	Math Lab 50 Min.	Lecture 50 Mins.	Math Lab 50 Min.	Lecture 50 Min.

Units

Instructor: 3

Student : 4

MODE B

Monday	Tuesday	Wednesday	Thursday	Friday
Lecture 50 Min.	Lecture 35 Min. ------- Quiz 15 Min.	Math Lab 50 Min.	Lecture 50 Min.	Lecture 35 Min. ------- Quiz 15 Min.

Monday	Tuesday	Wednesday	Thursday	Friday
Math Lab 50 Min.	Lecture 50 Min.	Lecture 35 Min. -------- Quiz 15 Min.	Math Lab 50 Min.	Lecture 50 Min.

Selected Thursdays
EXAM 50 Mins.

Monday	Tuesday	Wednesday	Thursday	Friday
Lecture 35 Min. ------- Quiz 15 Min.	Math Lab 50 Min.	Lecture 50 Min.	Lecture 35 Min. ------- Quiz 15 Min.	Math Lab 50 Min.

Units - Same as Above

Instructor : 425 Mins.
per 15 days

440 Mins.
EXAM periods

PROPOSAL TO PROVIDE A 4-UNIT TRIGONOMETRY COURSE,
3 LECTURE HOURS & 2 HOURS OF PROBLEM SESSIONS WEEKLY

RATIONALE FOR RESTRUCTURING TRIGONOMETRY AT CSM

Trigonometric functions provide a departure for students from the customary polynomial functions encountered in elementary and intermediate algebra courses. About the only challenging function found in the traditional INTERMEDIATE ALGEBRA course is the absolute value function. Thus, students come to TRIGONOMETRY having learned about the fundamental operations on, and the properties of polynomial and rational functions in their previous course. Yet, they still come woefully ill-prepared for the study of trigonometric functions. JAMES L. JACKSON captures this ill-preparedness well and describes it, and its precedents eloquently in his paper following this statement and proposal.

The satisfactory completion of MATH 130 is a necessary requirement for every subsequent mathematics course at CSM, excepting MATH 151-152, MATH 162, and MATH 200. Even in those latter courses, MATH 130 can greatly expand the student's experiences.

In trigonometry, the student must rely heavily on his understanding of, and application of definitions. The task of applying definitions, coupled with many students' inadequate algebraic skills and convictions, causes students to be mired in difficulty in the trigonometry course from the beginning. Success in MATH 130 heavily relies on the successful application of definition. However, our students also display lack of understanding of the generalized function concept, and an inability to determine function domain and range, as well as restrictions inherent therein, from the nature of definition.

It is a GIVEN that the student must know and understand the trig function definitions, the core of the course. Further, he must demonstrate ability to use these definitions in applications. There is widespread dissatisfaction with the degree to which this goal is achieved at this time in trig classes. The current time-frame for this aspect of MATH 130 is inadequate. Recommended, therefore, are problem sessions in combination with a regular lecture series. Two required problem sessions will provide much-needed extra time and an altered setting to accomplish goals. This environment

cannot but motivate the student to do assigned problems and remedial work, if qualified tutors are present for consultation and/or guidance. Moreover, the student develops the habit of study, of doing mathematics rather than participating as a spectator.

A study of our recent grade reports at CSM indicates that Trigonometry ranks among the highest of attrition-ridden courses in the mathematics program. The record shows that all instructors, without exception, encounter high attrition in this course. Beyond that, staff witness a struggle to the finish by the students who stick it out.

It is, therefore, proposed that the trigonometry course be restructured, (1) by further demands in material - both in quality and kind - and (2) by restructure of the time-frame for the course's completion. The course will consist of two components : lecture and problem session. LECTURE will be three hours weekly, Monday, Wednesday Friday. PROBLEM SESSIONS will be two hours weekly - Tuesday and Thursday, at the same hour as the lecture hour.

COURSE FORMAT

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
Lecture 50 Minutes	Quiz 15 Minutes --- Problem Session 35 Minutes	Lecture 50 Minutes	Quiz 15 Minutes --- Problem Session 35 Minutes	Lecture 50 Minutes

This schedule is equivalent to 4 student contact hours :

$$\frac{150(\text{Minutes, Lecture}) + 30(\text{Minutes, Quiz}) + 70/3(\text{minutes, Problems})}{50 \text{ Minutes (1 student contact hour)}}$$

$$= \frac{203}{50} \simeq 4 \text{ (Units)}$$

LECTURE FUNCTION : The instructor presents pertinent definitions, theorems and applications to elucidate or enrich. The instructor makes assignments he deems appropriate. At the outset, more time must be devoted to (1) information on the role of Problem Sessions - not only for attendance reporting, but to fulfill the function of these

problem sessions, as spelled out in the following paragraphs. These meetings will be operated by a math tutor who will receive specific instructions on attendance recording, homework reporting, quizzes to be administered and recording of corresponding grades from MATH 130 instructors.

FUNCTIONS OF PROBLEM SESSIONS

Homework assignments are an integral part of the course. Satisfactory completion of an assignment is defined to be 80% or better. To make homework assignments a necessary part of course completion as well as to provide maximum learning experiences therefrom, the proposed problem session component will fulfill the following functions:

1. Trig assignments are to be submitted to the math tutor no later than 72 hours following the time assignment was made, excepting Wednesday assignments which may be submitted no later than the following Tuesday problem session. Thus Monday assignments are to be submitted no later than the Thursday immediately following; Wednesday and Friday assignments are to be submitted no later than the Tuesday immediately following them.
2. Math 130 students are to report to the problem session area each Tuesday and Thursday. Here they check their homework that is due. If all are correct, they submit the assigned problems to the tutor for recording. They may now take the quiz to be given in conjunction with each homework assignment.
   If a student was unable to complete the prescribed assignment, he may seek help from the tutor. Upon satisfactory completion of the assignment, it is checked by the math tutor.
   When errors occur, they are to be identified by the student; he may seek help from the tutor in identifying his errors. He corrects errors before submitting assignment for recording purposes.
3. A quiz will be taken with relation to each homework assignment. It will be taken under supervision of the math tutor, and without consultation. If the student passes it with a grade of 70% or better, the grade is recorded by the tutor. The student may

continue working in the problem session area, or he may leave.

4. If a student does not score 70% or better, he will be given a second chance (to take the quiz). He is advised to study further in preparation for it. Upon taking this second try, the grade secured on this quiz becomes part of the student's record.
5. Math tutor will give the record of completed assignments and of quiz scores to the MATH 130 instructor periodically. Instructor uses them for student grading as he sees fit.
6. If a student misses a lecture, he is accountable for the study of the material covered, assignment that was made, and the quiz accruing. He may seek help during the subsequent problem session, upon availability of tutor time.
7. A further function of the problem sessions is for diagnostic test-taking, at the outset of new topics. Test will be administered before the first lecture in the new material in the problem-session center. If student's score on the diagnostic test is deemed inadequate, the student is provided with reference(s) to the needed remedial material.
8. Exams will be administered at designated intervals by the instructor. Instructor may choose to administer this exam during lecture hour or he may administer it during a problem session. Samples of exams may be filed in the center, should the instructor wish to make such available to students for study.
9. Problem sessions will support the learning process for A-level students, who too-often progress without sufficient challenge to their ability or imagination. The center, therefore, will have supplementary problems for such students. These materials will have been selected by the instructor. The student who does not need to do remedial work and wishes to get extra credit may do so by working these selected problems. Tutorial help may not be sought for such endeavors.

HOW THIS PROPOSAL AFFECTS PERSONNEL

Instructors of MATH 130 will provide three contract hours per week in the lecture segment. While instructors will have to devote some time to synchronizing problem session materials and activities, they will be saved some time, with

properly-trained math tutors who will grade, collect, and record papers and quizzes.

This proposal presumes the availability of one math tutor per MATH 130 section, as well as overall supervision of the center.

The provision of the latter will restore the long overdue Mathematics Learning Center, where students from MATH 130, MATH 110, as well as from other Math classes, long neglected, can work to achieve their common endeavor - learning mathematics.

JAMES L. JACKSON'S VIEWS ON DEFICIENCIES OF STUDENTS OF MATH 130

It is widely supposed that the failure of so many students to succeed in trigonometry courses is that "their algebra is poor". While it is difficult, even impossible, sensibly to disagree with this appraisal, it is, nevertheless, too general to be of any use whatever as an hypothesis to account for this failure. It is necessary to determine what it is that the students learned, or failed to learn, in their algebra courses which creates the block over thich they stumble. It may turn out to be that that which we seek is too subtle for us to express clearly in simple terms. Indeed, the term "mathematical maturity" was coined because others before us have found it so when confronted with similar tasks.

First of all, it needs to be understood at the outset that there are some students who succeed quite well in their trigonometry courses having "had" the same algebra courses as other students who do not do well. For this reason it is easy, and even justified, to blame the failure on the bad genes or the underpriviledged cultural background of the student who does not do well. This attribution of responsibility is especially justifiable if most of the students involved succeed while only a few fail as was the case in typical trigonometry classes a generation or so ago. But with the society of students we deal with here it is the many who fail and the few who succeed. We therefore need to ask what it is that the "good" student learns in his/her algebra class that others do not. Or, perhaps, what the "good" student overcomes, or unlearns, in the course of learning trigonometry.

It is my opinion that algebra courses are typically taught in such a way that students can pass often with quite good grades having learned little except the manipulative skills of algebra. Moreover, these skills have been learned by techniques and mnemonic devices that have little or no relation to the mathematical concepts that underlie these manipulations. I think that this is due to at least these two notions : First, that these skills are of primary importance in learning algebra and second, that any device by which a student can be trained to give the correct prescribed response to a given algebraic stimulus is a proper device for a teacher to use.

I concur with the view expressed by the first of these with the qualification that "primary importance" does not mean to the exclusion of all else. But I believe the other to be a misguided notion. Indeed, I believe that as a consequence of this kind of training, students develop a misconception of the nature of mathematical study as a body of knowledge whose ideas are logically interconnected. Furthermore, I believe that learning trigonometry can be made considerably easier for most of our students if they can be brought to perceive its study in this way. But, because they have successfully passed a course in algebra, many of them persist in the view that trigonometry can be successfully studied as a sequence of techniques

learned to solve a set of distinct and scarcely related problems just as they had done with their algebra course.

That the student should conceive of the study of mathematics as a collection of techniques to solve a set of prescribed problems is no surprise if the student was presented algebra in the typical way. Unless a teacher is careful, even the best of algebra courses can degenerate into a recital of "rules" since the persistent threads of thought are elementary and easily overlooked in the parade of topics: Special Products and Factoring, Simplifying Fractions, Linear Equations, Roots and Radicals, etc. But trigonometry, on the other hand, consists simply in the study of the elementary properties of a small handful of functions each of which is so closely related to the others that each of the others can be defined from just one or two of them. A list of the topics in this course (Definitions of the Trigonometric Functions, Graphs of the Trigonometric Functions, Identities Involving the Trigonometric Functions, Equations Involving the Trigonometric Functions, etc.) seems to suggest, at least to one who has been trained to look for such things, that all these topics may be connected somehow and perhaps the most significant persistent thread here is the notion of the trigonometric functions themselves!

Whenever I hear that students fail in trigonometry because "their algebra is bad" I assume that what is meant is that they don't know how to solve quadratic equations, for example, or factor a difference of two squares or simplify a complex fraction and I have the disquieting feeling that the implied cure is more emphasis on drill with mnemonic devices and specious analogies as a substitute for honest confrontation of mathematical ideas. I believe, on the other hand, that while drill is very important, the foundations for the techniques drilled should be well founded in honest experience and sound logic whenever possible and these underlying ideas should be made persistent and recurring themes throughout an algebra course. It is in this way that I believe that a student can be best prepared for trigonometry.

I believe that the greatest problem that my students encounter is in understanding the simplest and the most basic notions of mathematics. I am asked for example, "Does it matter if I write

$$\sin x \cos y + \sin y \cos x$$

instead of

$$\sin x \cos y + \cos x \sin y$$

in the formula for $\sin(x + y)$?" And simple substitutions as a (y) for x in

$$\sin(a - x) = \cos x$$

to get

$$\cos(a - y) = \sin y$$

seem to cause considerable difficulty for some students. To what can we attribute such poor conceptions if not "bad algebra"? But solving more quadratic equations will not help unless the substitutions involved in a general factoring form or in the quadratic formula are meticulously pointed out.

Another ploy that might work to lessen the extent of the gap between algebra and trigonometry is the employment of such notation of f(x) and sqr(x) in the manipulative exercises so that the students might be confronted with expressions to simplify like

$$sqr(x)\{sqr(x) - x\}$$

and

$$\{f(x) + 4\}\{f(x) - 6\}$$

Such expressions as

$$(x-3)^2 + 3(x-3) + 2,$$

which the student is asked to factor, help in this regard but do not go far enough.

Since such exercises are at present not given in typical algebra courses (except possibly for the last of these) for the reasons alluded to earlier, it seems to me that the next best thing would be to include such questions on diagnostic tests given to students as a "preparedness" test. Such a test cannot, of course, be a "standardized" test and such a test with such "non-standard" questions must be thoroughly re-capitulated for the students with full explanations including the mention of every instance of the use of commutativity of multiplication and the like. (After doing precisely this for a short calculus class recently, a student came to me asking if there was a course which would better prepare her for the calculus course since "I understood scarcely anything you have been saying...", she said.)

I am not sure that I have put my finger on the meaning of "mathematical maturity" in this context but I believe that I have found at least a partial truth with respect to student failure in trigonometry. And until this hypothesis is tested and found wanting it will be difficult to convince me that my notions are mistaken.

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